Name: ____________________________

Instructor: ________________________

Math 10550, Practice Exam III
November 15, 2023

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- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 11 pages of the test.

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3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
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9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

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Multiple Choice	______________
11.	______________
12.	______________
13.	______________
Total	______________

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Multiple Choice

1.(6 pts.) The slant asymptote of $y = \dfrac{2x^4 + x^3 + 5}{x^3 - 3x^2 + 2}$ is given by

(a) $y = 2x - 5$ (b) $y = 2x + 4$ (c) $y = 2x + 7$

(d) $y = x + 4$ (e) There are no slant asymptotes.

Solution: Using long division, we find that

$$\frac{2x^4 + x^3 + 5}{x^3 - 3x^2 + 2} = (2x + 7) + \frac{21x^2 - 4x - 9}{x^3 - 3x^2 + 2}$$

2.(6 pts.) Evaluate $\displaystyle\lim_{x \to -\infty} \frac{\sqrt{4x^6 + 5}}{x^3 + 1}$.

Solution: We first divide through by x^3, in the numerator using the fact that $\sqrt{x^6} = -x^3$ for $x < 0$:

$$\frac{\sqrt{4x^6 + 5}}{x^3 + 1} = -\frac{\sqrt{4 + 5/(x^6)}}{1 + 1/(x^3)}.$$

As x goes to $-\infty$, the numerator goes to $\sqrt{4} = 2$. As x goes to $-\infty$, the denominator goes to 1. Hence the answer is -2.

(a) 3/2 (b) 6 (c) 2 (d) -2 (e) 4

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3.(6 pts.) If we want to use Newton's method to find an approximate solution to

$$\cos(x) - x = 0$$

with initial approximation $x_1 = \frac{\pi}{2}$, what is x_2?

(a) π (b) $\frac{\pi}{4}$ (c) 0 (d) $\frac{3\pi}{4}$ (e) $\frac{\pi}{2}$

Solution: Take $f(x) = \cos(x) - x$. Using that $f'(x) = -\sin(x) - 1$ and the formula $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$, we have

$$\begin{aligned} x_2 &= \frac{\pi}{2} - \frac{-\pi/2}{-2} \\ &= \pi/2 - \pi/4 \\ &= \pi/4 \end{aligned}$$

4.(6 pts.) A bug being chased by a kitten (both moving in a straight line) enters a kitchen with velocity 1 ft/sec, and accelerates at $\frac{2}{\sqrt{t}}$ft/sec^2. How fast is the bug moving 9 seconds later.

(a) 5 ft/sec (b) 37 ft/sec (c) 13 ft/sec

(d) 4 ft/sec (e) 7 ft/sec

Solution: Since acceleration is given by $a(t) = 2t^{-1/2}$ and the derivative of velocity is acceleration, we know that after integrating acceleration, the velocity is given by $v(t) = 4t^{1/2} + C$ for some constant C. We are also given that the initial velocity is 1 so that $v(0) = C = 1$. Thus, $v(t) = 4^{1/2} + 1$. Thus, $v(9) = 12 + 1 = 13$.

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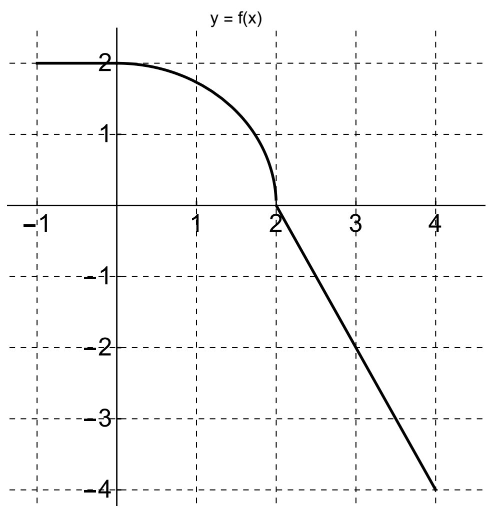
5.(6 pts.) The graph shown below is that of $f(x)$ for $-1 \le x \le 4$ where

$$f(x) = \begin{cases} 2 & \text{if} -1 \le x \le 0 \\ \sqrt{4-x^2} & \text{if} 0 < x \le 2 \\ 4-2x & \text{if} 2 \le x \le 4 \end{cases}$$

Which of the following equals $\int_{-1}^{4} f(x)dx$?



(a) π (b) $2\pi - 2$ (c) $6 + \pi$

(d) 0 (e) $\pi - 2$

Solution: From -1 to 0, the function is constant with output 2. So the area under the curve is given by the area of a rectangle with base 1 and height 2. The area of such a rectangle is 2. From 0 to 2, we see that the function traces out the upper left portion of a circle with radius 2 centered at $(0, 0)$ and hence, we have that the area is given by $\frac{1}{4}\pi 2^2 = \pi$. From 2 to 4, we see that the curve traces the hypotenuse of a right triangle with base 2 and height 4. The area of such a triangle is $\frac{1}{2}(2)(4) = 4$. However, since function is under the $x-$axis from 2 to 4, we need to account for this by subtracting the area of the triangle. So the integral is evaluated to be $2 + \pi - 4 = \pi - 2$.

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6.(6 pts.) If $f(x) = \int_{x^3}^{1} \sqrt{1+\sin(t)}\, dt$, then $f'(x) =$

(a) $-3x^2\sqrt{1+\sin(x^3)}$ (b) $\sqrt{1+\sin(x)}$ (c) $-\sqrt{1+\sin(x^3)}$

(d) $\sqrt{1+\sin(x^3)}$ (e) $3x^2\sqrt{1+\sin(x^3)}$

Solution: Setting $y = x^3$, we rewrite as $f(y^{1/3}) = \int_y^1 \sqrt{1+\sin(t)}\, dt$. Then we have

$$-f(y^{1/3}) = \int_1^y \sqrt{1+\sin(t)}\, dt.$$

Taking the derivative of both sides, we see via the chain rule and the Fund. Thm. of Calc. that

$$\begin{aligned} -f'(y^{1/3}) * 1/3y^{-2/3} &= \sqrt{1+\sin(y)} \\ \Rightarrow f'(y^{1/3}) &= -3y^{2/3}\sqrt{1+\sin(y)} \\ \Rightarrow f'(x) &= -3x^2\sqrt{1+\sin(x^3)} \end{aligned}$$

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7.(6 pts.) Evaluate $\int_0^4 |x-2|dx$.

(a) 0 (b) 2 (c) 6 (d) -2 (e) 4

Solution: We see that $x-2$ is positive when $x>2$ and $x-2$ is negative when $x<2$. We compute

$$\int_0^4 |x-2|dx = \int_0^2 (2-x)dx + \int_2^4 (x-2)dx = [2x - \frac{x^2}{2}]_0^2 + [\frac{x^2}{2} - 2x]_2^4 = (4-2) + (8-8) - (2-4) = 4.$$

8.(6 pts.) Evaluate

$$\int \frac{\sin(x^{2/3})}{\sqrt[3]{x}}dx.$$

(a) $-\cos(x^{2/3}) + C$

(b) $\frac{3}{2}\cos(x^{2/3}) + C$

(c) $-\frac{2}{3}\cos(x^{2/3}) + C$

(d) $-\frac{3}{2}\cos(x^{2/3}) + C$

(e) $\cos(x^{2/3}) + C$

Solution: Setting $u = x^{2/3}$, we have $du = \frac{2}{3}x^{-1/3}$ and

$$\int \sin(u)\ du = \frac{2}{3}\int \frac{\sin(x^{2/3})}{\sqrt[3]{x}}dx$$

$$\Rightarrow -\cos(u) + C = \frac{2}{3}\int \frac{\sin(x^{2/3})}{\sqrt[3]{x}}dx$$

$$\Rightarrow \int \frac{\sin(x^{2/3})}{\sqrt[3]{x}}dx = -\frac{3}{2}\cos(x^{2/3}) + C.$$

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9.(6 pts.) Evaluate

$$\int_1^2 \frac{\sec^2(\sqrt{x})}{\sqrt{x}} dx.$$

(a) $\dfrac{\tan(2)}{2} - \dfrac{\tan(1)}{2}$

(b) $2\tan(\sqrt{2}) - 2\tan(1)$

(c) $\dfrac{\tan(\sqrt{2})}{2} - \dfrac{\tan(1)}{2}$

(d) $\tan(\sqrt{2}) - \tan(1)$

(e) $2\tan(2) - 2\tan(1)$

Solution: First we look for an antiderivative. We try the function $F(x) = tan(x^{1/2})$. We see that $F'(x) = \dfrac{sec^2(x^{1/2})}{2x^{1/2}}$. Thus, our initial guess is incorrect, but we see that $F(x) = 2tan(x^{1/2})$ is an antiderivative for the integrand. Applying the FTC, we see that the answer is $F(2) - F(1) = 2tan(2^{1/2}) - 2tan(1)$.

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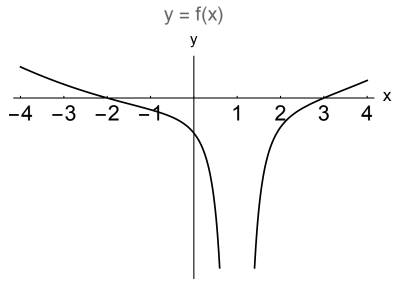
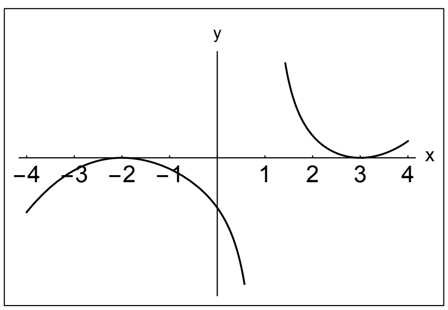
10.(6 pts.) The graph of $f(x)$ is shown below:



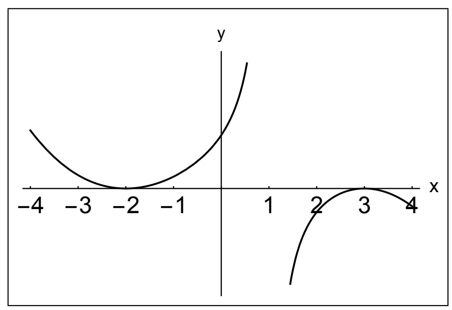
which of the following gives the graph of an antiderivative for the function $f(x)$?

Solution Denote $F(x)$ to be one antiderivative of $f(x)$. By definition, $F'(x) = f(x)$. So $F(x)$ is increasing in $[-4,-2]$ and $[3,4]$; decreasing in $[-2,1) \cup (1,3]$.

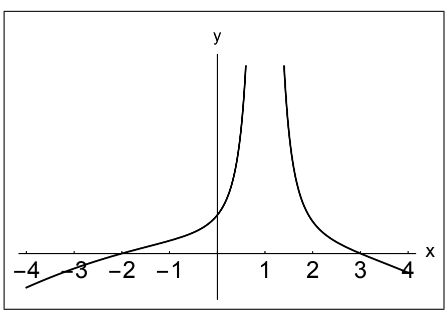
(a)



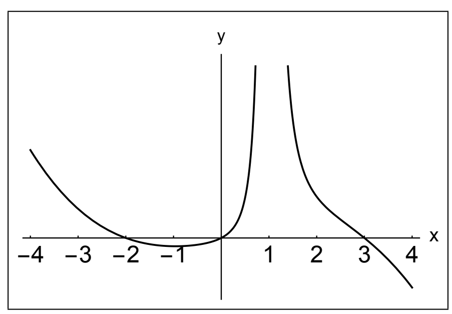
(b)



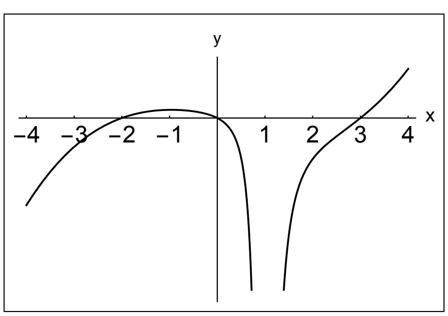
(c)



(d)



(e)



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Partial Credit

You must show your work on the partial credit problems to receive credit!

11.(13 pts.) A page of a book is to have a total area of 150 square inches, with 1 inch margins at the top and sides, and a 2 inch margin at the bottom. Find the dimensions in inches of the page which will have the largest print area.

Let x denote the total width and y denote the total height. So the width of the printed area is $x-2$ and the height of the printed area is $y-3$. Then the total area of the page can be expressed as

$$A_{total} = xy.$$

We are given that $A_{total} = 150$, so $y = 150/x$. We wish to maximize

$$A_{print} = (x-2)(y-3) = (x-2)\left(\frac{150}{x} - 3\right) = 156 - 3x - \frac{300}{x}.$$

Differentiating with respect to x and finding critical points gives

$$A'_{print}(x) = -3 + \frac{300}{x^2} = 0$$

so we must have $300 - 3x^2 = 0$, i.e. $x^2 = 100$. So $x = 10$ inches.

Using the first derivative test shows that 10 is indeed a maximum. For $x < 10$, $A'_{print} > 0$, and for $x > 10$, $A'_{print} < 0$.

$y = 150/x$, so we have $y = \dfrac{150}{10} = 15$. Therefore the page which maximizes the printed area has the dimensions 10 inches by 15 inches.

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12.(13 pts.) A particle is moving in a straight line with acceleration

$$a(t) = 4\left(t^2 - \frac{1}{3}\right) \text{ ft}/s^2,$$

where distance is measured in feet and time in seconds. The initial velocity of the particle is $v(0) = 0$ ft/s and the initial position of the particle is $s(0) = 0$.

(a) Find the velocity of the particle at time t (i.e. find $v(t)$).

Solution: $v(t) = \int a(t)\ dt = \frac{4}{3}t^3 - \frac{4}{3}t + C$. Moreover, since $v(0) = 0$, solving for C we get $C = 0$.

(b) Find the position of the particle at time t (i.e. find $s(t)$).

Solution: $s(t) = \int s(t)\ dt = \frac{1}{3}t^4 - \frac{2}{3}t^2 + C$. Moreover, since $s(0) = 0$, solving for C we get $C = 0$.

(c) Find the values of t for which $v(t) = 0$ on the interval $[0, \infty)$.

Solution: $v(t)\frac{4}{3}t(t^2 - 1) = 0$ implies $t = \pm 1$ or $t = 0$. Hence, $t = 0$ and $t = 1$ are the only values for t in $[0, \infty)$ where $v(t) = 0$.

(d) Find the <u>distance</u> travelled by the particle on the time interval $0 \leq t \leq 2$.

Solution: Observe that $v(t) < 0$ for $0 \leq t \leq 1$ and $v(t) > 0$ for $1 \leq v(t) \leq 2$. Total distance traveled is then:

$$\begin{aligned}\int_0^2 |v(t)|\ dt &= \int_0^1 -v(t)\ dt + \int_1^2 v(t)\ dt \\ &= [-\frac{1}{3}t^4 + \frac{2}{3}t^2]_0^1 + [\frac{1}{3}t^4 - \frac{2}{3}t^2]_1^2 \\ &= -\frac{1}{3} + \frac{2}{3} + \frac{16}{3} - \frac{8}{3} - \frac{1}{3} + \frac{2}{3} = \frac{10}{3}\end{aligned}$$

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13.(14 pts.) Evaluate the definite integral shown below using right endpoint approximations and the limit definition of the definite integral

$$\int_0^2 \frac{x}{2}\,dx$$

$\left(\text{Note: } 1+2+3+\cdots+n = \sum_{i=1}^{n} i = \dfrac{n(n+1)}{2}.\right)$ Verify your answer using the fundamental theorem of calculus. **Solution:** We compute $\Delta x = 2 - 0/n = 2/n$ and $x_i = i\Delta x = 2i/n$. Then

$$\int_0^2 \frac{x}{2}\,dx = \lim_{n\to\infty}\sum_{i=0}^{n} f(x_i)\Delta x = \lim_{n\to\infty}\sum_{i=0}^{n}\frac{2i}{2n}\left(\frac{2}{n}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n^2}\sum_{i=0}^{n} i = \lim_{n\to\infty}\frac{2}{n^2}\left(\frac{n(n+1)}{2}\right) = \lim_{n\to\infty}\frac{n^2+n}{n^2} = 1.$$

Using the FTC we get

$$\int_0^2 \frac{x}{2}\,dx = \left.\frac{x^2}{4}\right|_0^2 = \frac{4}{4} - \frac{0}{4} = 1.$$

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Multiple Choice ____________

11. ____________

12. ____________

13. ____________

Total ____________